EE215 – FUNDAMENTALS OF ELECTRICAL ENGINEERING

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WEEK 9
FIRST ORDER CIRCUIT RESPONSE II & SECOND ORDER CIRCUIT RESPONSE I

May 28th , 2010

FIRST ORDER CIRCUITS RESPONSE

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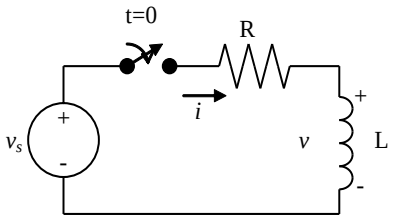
QUESTIONS TO ANSWER

- First order circuits
  - How to analyze and solve the step response of RL and RC circuits?
  - How to solve an electric circuit with multiple switches?
- Second order circuits
  - How to analyze and solve the natural response of RLC circuits with different voltage responses?

STEP RESPONSE FOR RL CIRCUIT (1)

- Now let's look at a circuit with an inductor instead of a capacitor. We'll start with the simplest possible circuit:

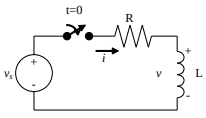


STEP RESPONSE FOR RL CIRCUIT (2)

- From KVL:

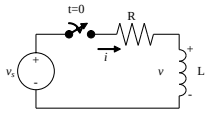
- Compare this to

- from the RC circuit. It's not *quite* the same - but we can make it closer…

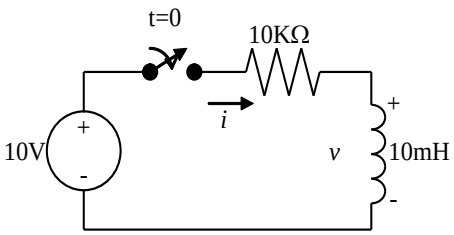


STEP RESPONSE FOR RL CIRCUIT (3)

- NOW this is the same equation - which means it must have the same form of solution



EXAMPLE OF STEP RESPONSE FOR RL CIRCUIT (1)



Find $i(t)$ for $0 < t < \infty$!

EXAMPLE OF STEP RESPONSE FOR RL CIRCUIT (2)

1. Find the initial condition.

2. Find the steady state solution.

EXAMPLE OF STEP RESPONSE FOR RL CIRCUIT (3)

3. Find the time constant

4. Write the solution,

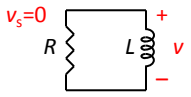
EXAMPLE OF STEP RESPONSE FOR RL CIRCUIT (4)

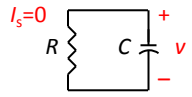
5. Plug the two boundary values (the values at t = 0 and t = ∞) into the solution to find *A* and *B*. Use the steady state value first!

6. Make sure you answered the question being asked!

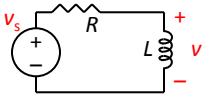
GENERAL SOLUTION FOR NATURAL AND STEP RESPONSE

Natural Response





Step Response



Equation

Time constant

$t \rightarrow \infty$

GENERAL SOLUTION FOR NATURAL AND STEP RESPONSE (4)

- … in words:

$$x(t) = x_\infty + \left(x(t_0) - x_\infty\right) \cdot e^{-\frac{t-t_0}{\tau}}$$

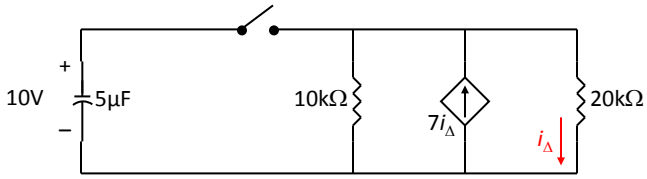
The unknown variable as a function of time	=	The final value of the variable	+	The difference between initial and final value	·	Exponential decay

SEQUENTIAL SWITCHING (1)

- More than one switching operation in sequence.
- Consequence:

- Approach:
  -
  -
  -

SEQUENTIAL SWITCHING (2)



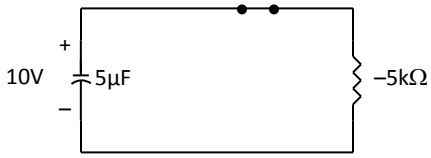
1. Find Thévenin equivalent w.r.t. C:

SEQUENTIAL SWITCHING (3)

- 2. Find (unbounded) response:
  - For $t \geq 0$:
  - Solution:

SEQUENTIAL SWITCHING (4)

- 3. At what time will circuit fail if capacitor breaks down at 150V?

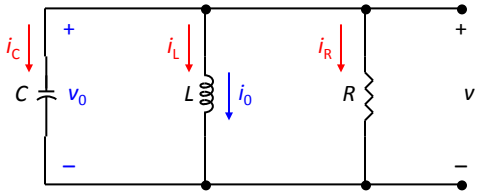


NATURAL AND STEP RESPONSES OF RLC CIRCUITS

- Circuits with two energy storage elements are called *second order circuits*, because they give rise to second order linear differential equations. The most interesting behavior of these circuits happens in the RLC circuit, with an inductor and a capacitor.
- Second order circuits are solved in much the same way as first order circuits, by writing the form of the solution and then finding the coefficients from initial and steady state conditions. It's a little more complicated.

NATURAL AND STEP RESPONSES OF RLC CIRCUITS

- Example: parallel RLC circuit



- Natural response:
  - v_0 :
  - i_0 :

HOW DOES SOLUTION LOOK LIKE?

Special cases:

1. $C = 0$ (and $v_0 = 0$) ➔
2. $L = \infty$ (and $i_0 = 0$) ➔
3. $R = \infty$
   -
   -
   -
   -

GENERAL SOLUTION OF THE PARALLEL RLC CIRCUIT (1)

- Node voltage method:
- Differentiate:
- Divide by C and rearrange:
- Ordinary second-order differential equation
- ➔

GENERAL SOLUTION OF THE PARALLEL RLC CIRCUIT (2)

- Assumption: solution is of the form $v(t) = A\,e^{st}$ with A, s unknown constants
- Plug into equation above:
- Rearrange:
- In general, $A \neq 0$, $e^{st} \neq 0$ thus
- "Characteristic Equation"

SOLUTION TO CHARACTERISTIC EQUATION (1)

- Often we use the following parameters:
  - Neper frequency:
  - Resonant radian frequency:
  - Characteristic roots:

SOLUTION TO CHARACTERISTIC EQUATION (2)

- Three cases:
  - $\alpha^2 - \omega_0^2 > 0$:
  - $\alpha^2 - \omega_0^2 = 0$:
  - $\alpha^2 - \omega_0^2 < 0$:
- Solutions:

  with some parameters

- In summary, the natural response of the parallel RLC circuit is

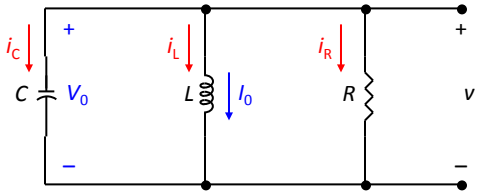
SOLUTION TO CHARACTERISTIC EQUATION (3)

- The voltage is of the form
- Determine A_1 and A_2:
  - Initial condition of circuit poses two constraints
- We need to solve:

SOLUTION TO CHARACTERISTIC EQUATION (4)

- KCL:
- Thus we have a linear system of 2 equations with 2 unknowns:
- This system can be solved for A_1 and A_2.
- Summary:
  1.
  2.
  3.
  4.

PARALLEL RLC CIRCUIT EXAMPLE/OVERDAMPED



- $R = 200\Omega$, $L = 50\text{mH}$, $C = 0.2\mu\text{F}$, $V_0 = 12\text{V}$, $I_0 = 30\text{mA}$

PARALLEL RLC CIRCUIT EXAMPLE/OVERDAMPED

- Initial currents in each branch:
  - ➢
  - ➢
  - ➢
- Initial dv/dt :
  - ➢
- Determine A_1 and A_2:
  - ➢
  - ➢

PARALLEL RLC CIRCUIT EXAMPLE/OVERDAMPED

$$v(t) = -14e^{-5000t} + 26e^{-20000t} \quad \text{for} \quad t \geq 0$$